Statistical analysis of fading power vectors for real-time atmospheric channel emulation

SPIE.

Yousef K. Chahine, Evan J. Katz, Brian E. Vyhnalek, and Sarah A. Tedder
NASA Glenn Research Center

Introduction

A statistical analysis of atmospheric fading is presented with the goal of producing a practical engineering model for the fading channel suitable for generating synthetic fade vectors in real time for long-duration receiver testing with channel interleaving.

The work is focused on **turbulence-induced fade** including

(1) an engineering model for **aperture-averaged scintillation over slant-path space-to-ground links**, and

(2) fluctuations in the **instantaneous coupling efficiency** for mode-limited receivers coupling to **few-mode fibers**.

Fading channel models based on Gaussian processes

The motivation of this work is to develop a

simplified model for **continuous, real-time generation of fading power vectors** for **controlled, long-duration tests with channel interleaving**.

To completely specify channel statistics, we model the received power signal $P(t)$ based on ergodic Gaussian processes **fully determined** by the **probability density function** and temporal **power spectral density** $\Phi(\omega)$.

Instantaneous coupling efficiency fluctuations for few-mode fibers

Probability density function and **temporal power spectrum** calculated from **Monte Carlo simulation** with **Kolmogorov phase screens**.

Passive few-mode fiber coupling efficiency with M guided modes depends on

Coupling parameter:

$$\eta_0 = \frac{M}{(1+(D/r_0)^{5/3})^{6/5}}.$$

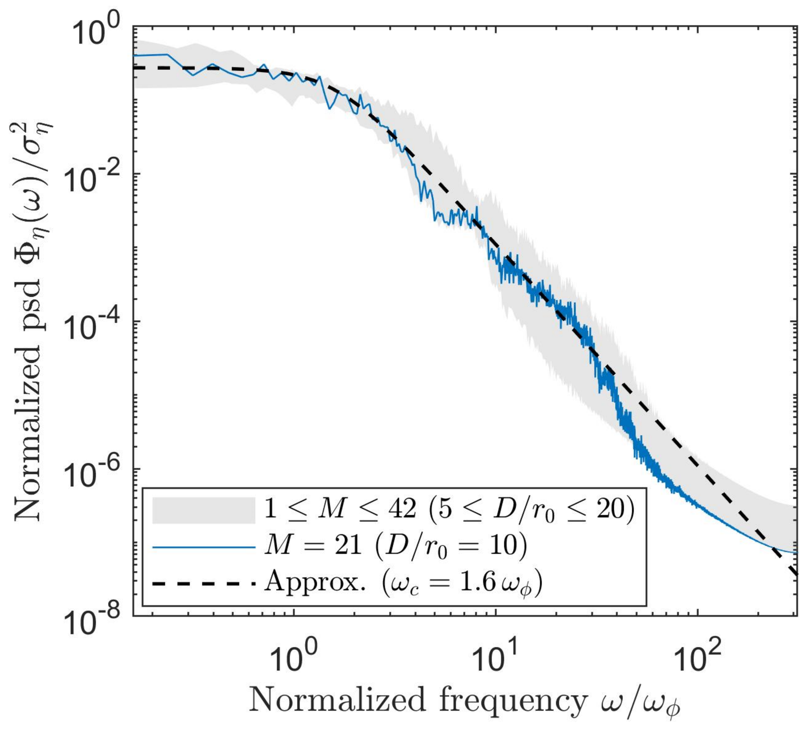

Figure 6. **Temporal power spectrum** from simulations with $1 \le M \le 42$.

$$\Phi(\omega) \propto \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^\alpha} \qquad \alpha = 3$$

Cutoff determined within factor of 3 for $1 \le M \le 42$

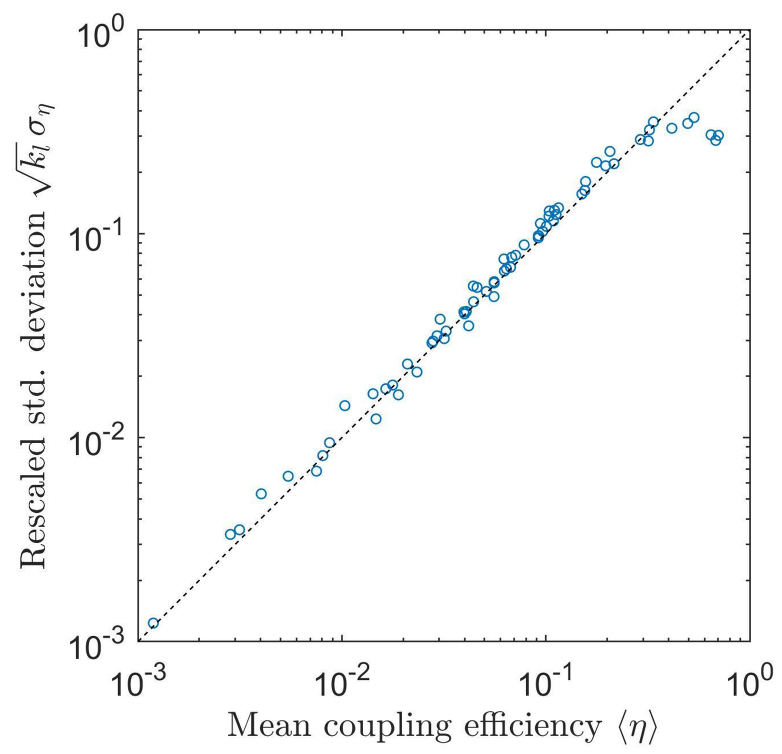

Figure 4. For $\eta_0 < 0.3$, **normalized variance** approximated by $1/k_l$ where $k_l = l_{\max} + 1$ is the **number of azimuthal orders** guided by the fiber.

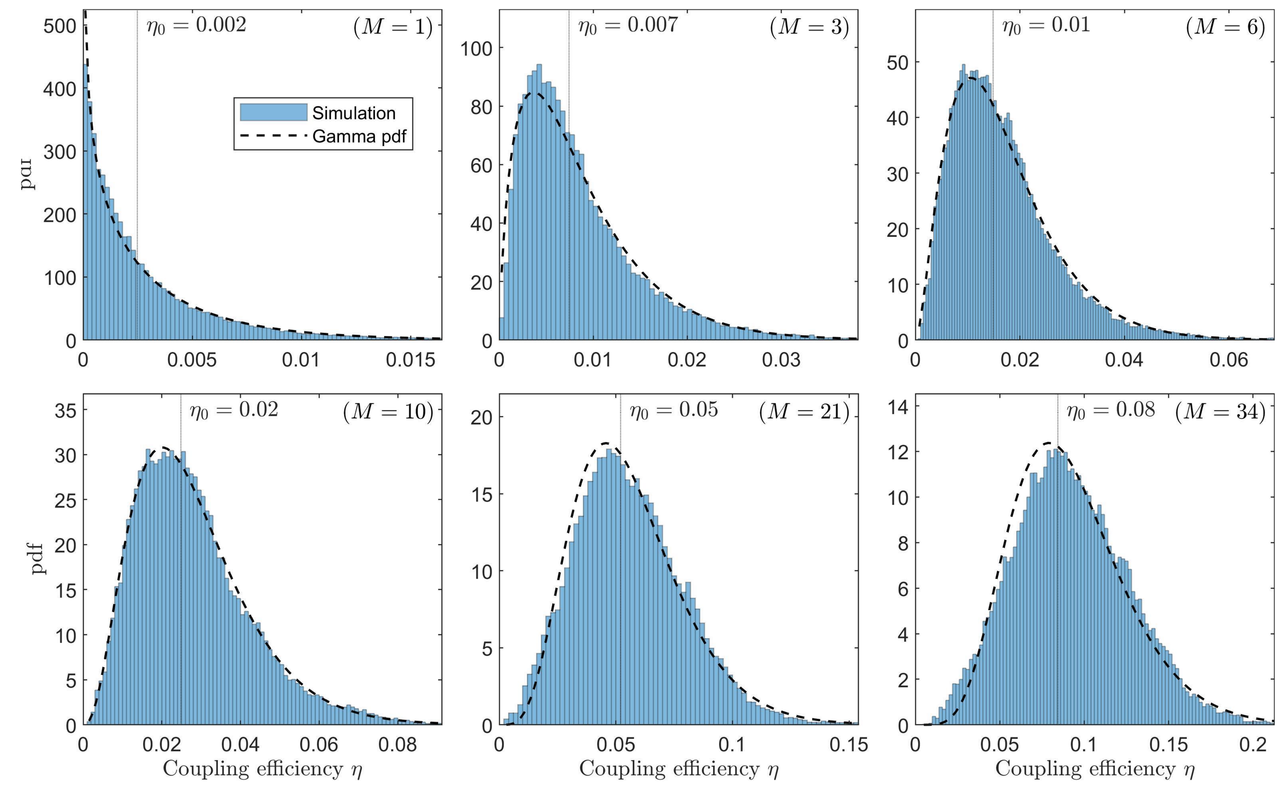

Figure 3. For $\eta_0 < 0.1$, (low-efficiency regime) the **probability density function** is approximated by a **gamma distribution** with k_l degrees of freedom.

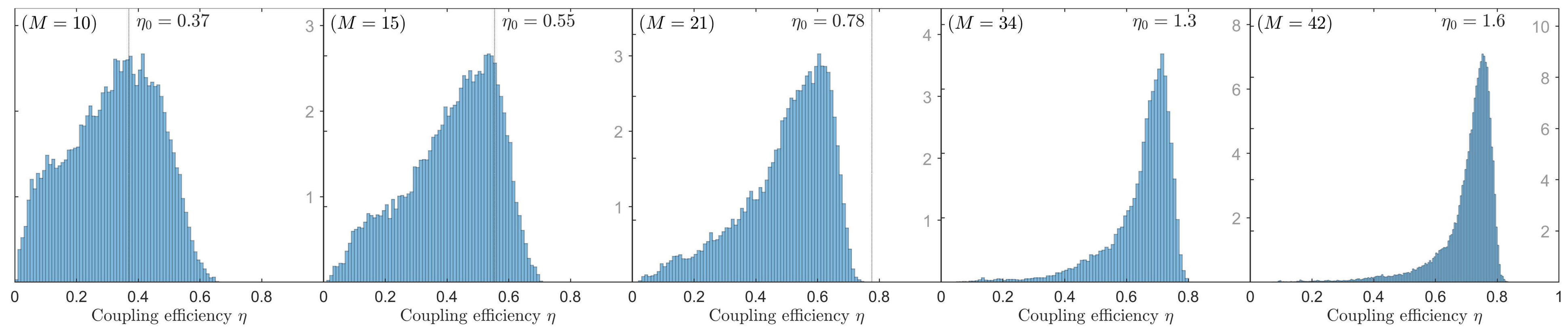

Figure 5. For $\eta_0 > 0.3$, (moderate to high-efficiency regime) the **probability density function** is estimated numerically from Monte Carlo simulation with $D/r_0 = 5$.

Engineering model for slant-path scintillation with aperture-averaging

Model reduction for slant-path links

Input Parameters:

$C_n^2(s)$ = structure constant profile

$V(s)$ = wind path profile

a = aperture radius

Yura's approximation

Effective path constants:

L_{eff} = effective scintillation path length

V_{eff} = effective scintillation rms wind

two-scale characteristic frequencies

Characteristic frequencies:

$\omega_F = V_{\text{eff}}/\sqrt{L_{\text{eff}}/k}$ = Fresnel frequency

$\omega_a = V_{\text{eff}}/a$ = aperture frequency

Single-scale reduction

Small-aperture $(\omega_F/\omega_a < 0.25)$ | Intermediate aperture $(0.25 < \omega_F/\omega_a < 3)$ | Large-aperture $(\omega_F/\omega_a > 3)$

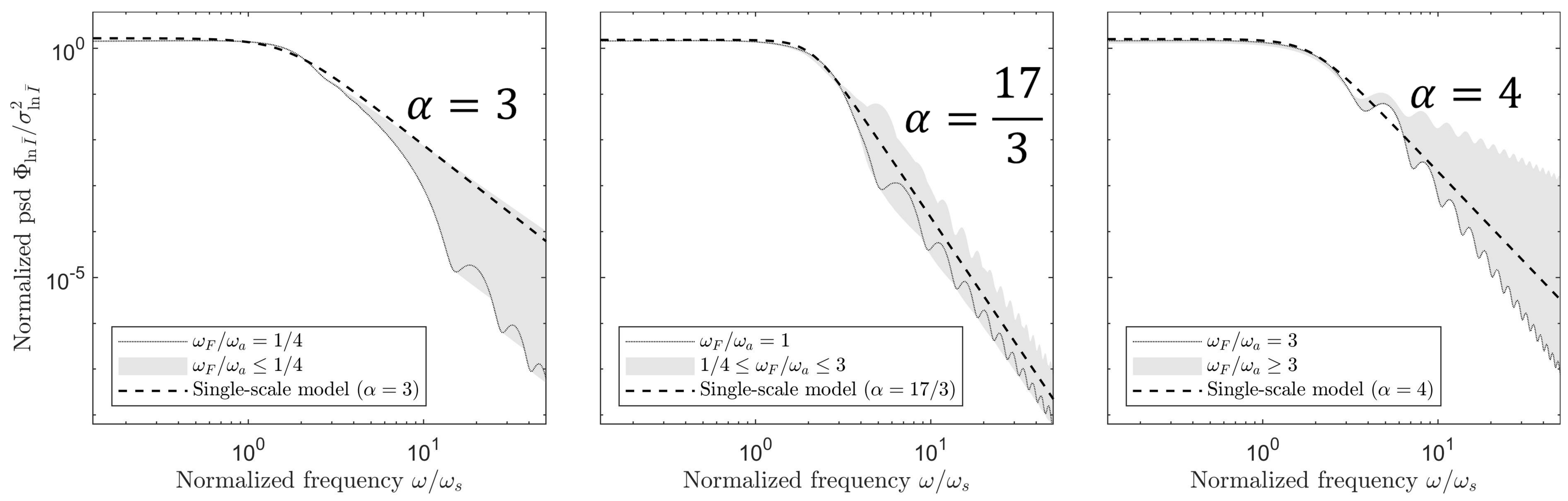

Figure 2. Single-scale models for temporal power spectral density of aperture-averaged irradiance fluctuations. Aperture size is categorized in 3 regimes where a single cutoff frequency and power law roll-off prevails.

Single-scale model for aperture-averaged scintillation PSD:

$$\Phi(\omega) \propto \frac{1}{1+\left(\frac{\omega}{\omega_c}\right)^\alpha}$$

Logarithmic slope

3 dB cutoff

$$\alpha = \begin{cases} 3 & \text{small aperture} \\ 17/3 & \text{intermediate aperture} \\ 4 & \text{large aperture} \end{cases}$$

$$\omega_c = 2.32^{1-3.86\alpha^{-2}} \frac{\omega_F}{\sqrt{1+(\omega_F/\omega_a)^2}}$$

Conclusions

Obtained practical engineering models suitable for synthesizing fading power vectors in real-time for continuous, long-duration testing of satellite ground receiver with channel interleaving.

Slant-path scintillation with aperture-averaging:

- Categorized into 3 regimes (small, intermediate, and large-aperture)
- Log-intensity PSD slope depends on regime, convenient approximations with Butterworth LPF (order 2 and 3).
- Single cutoff frequency (and correlation time) determined for general slant-path links using Yura's approximation

Few-mode fiber coupling fluctuations:

- Normalized **variance** inverse to **# guided azimuthal orders**
- Low-efficiency ($\eta_0 < 0.1$) PDF fit by **gamma distribution**
- Moderate/high-efficiency PDF requires further analysis (no closed form model determined for $\eta_0 > 0.25$)

This analysis enables a simplified testbed by including fluctuations in coupling efficiency directly in the fading channel model used to generate fade vectors without requiring real-time emulation of the turbulence-distorted optical wavefront arriving at the telescope.